Today’s Topics

- Inductance (Ch 32)
- Reminder of Faraday’s and Lenz’s Laws
- Mutual Inductance
- Self Inductance (Inductor L)
- Energy Stored in magnetic Field
- LR Circuit
Review: Faraday’s Law of Induction

Faraday’s Law in plain words: When the magnetic flux through an area is changed, an emf is produced along the closed path enclosing the area.

Quantitatively:

\[ \mathcal{E} = \frac{d\Phi_B}{dt} \]

Note the - sign
Review: Lenz’s Law

- Lenz’s law in plain words: the induced emf always tends to work against the original cause of flux change.

<table>
<thead>
<tr>
<th>Cause of $d\Phi_B/dt$</th>
<th>“Current” due to Induced $\mathcal{E}$ will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing B</td>
<td>produce $B$ in opposite dir.</td>
</tr>
<tr>
<td>Decreasing B</td>
<td>produce $B$ in same dir.</td>
</tr>
<tr>
<td>Relative motion</td>
<td>subject to a force in opposite direction of relative motions</td>
</tr>
</tbody>
</table>

Note: “Current” will not be actually produced if no circuit.
Examples of Lenz’s Law
Mutual Inductance

- For coupled coils:
  \[ \varepsilon_2 = -M_{12} \frac{dI_1}{dt} \]
  \[ \varepsilon_1 = -M_{21} \frac{dI_2}{dt} \]

Can prove (not in class):

\[ M_{12} = M_{21} = M \]

→ \( M \): mutual inductance

(unit: Henry)

\[ \varepsilon_2 = -M \frac{dI_1}{dt} \]
\[ \varepsilon_1 = -M \frac{dI_2}{dt} \]
Magnetically Coupled Coils (Transformers)

\[
\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}
\]

Question: Why use iron core?
Self Inductance

- When the current in a conducting device changes, an induced emf is produced in the opposite direction of the source current. \( \Rightarrow \) self inductance.

The magnetic flux due to self inductance is proportional to \( I \): \( \Phi_B = LI \)

\( \Rightarrow \) The induced emf is proportional to \( dI/dt \):

\[
\varepsilon_L = -L \frac{dI}{dt}
\]

\( L \): Inductance, unit: Henry (H)
Exercise: Calculate Inductance of a Solenoid

(Text example 32.1)
show that for an ideal solenoid:

\[ L = \frac{\mu_0 N^2 A}{\ell} \]

(see board)

(ideal case, recall Apere's Law)
Inductors

- Inductance is intrinsic to a conductive circuit.
- Two factors that determine the inductance:
  - Geometric configuration of the circuit
  - Filling of magnetic material.
- Specifically configured inductance devices (inductors) are very useful in electronic and electrical applications:
Charging and Discharging LR Circuit

Charging

Before

Transient

Stabilized

Dis-Charging

Before

Transient

After

\[ I = 0 \]

\[ I = i(t) \]

\[ I = I_{\text{max}} = \frac{\varepsilon}{R} \]
Energy in an Inductor

- When an inductor of inductance $L$ is carrying a current changing at a rate $dI/dt$, the power supplied is

$$P = I\epsilon = LI\frac{dI}{dt}$$

- The work needed to increase the current in an inductor from zero to some value $I$

$$W = \int dW = \int_0^I LI\,dI = \frac{1}{2}LI^2$$
Energy in a Magnetic Field

- **U= ½ LI²**
  - Solenoid: \( B = \mu_0 N/l I \) and \( L = \mu_0 N^2 A/l \)
  \[ U = \frac{1}{2} B^2/\mu_0 (A\ell) \]

- The energy is in the form of \( B \) field:
  - energy density: \( u_B = \frac{1}{2} B^2/\mu_0 \)
  (recall: \( u_E = \frac{1}{2} \varepsilon_0 E^2 \))

- Compare:
  - Inductor: energy stored \( U = \frac{1}{2} LI^2 \)
  \[ \Rightarrow \frac{1}{2} B^2/\mu_0 \]
  - Capacitor: energy stored \( U = \frac{1}{2} C(\Delta V)^2 \)
  \[ \Rightarrow \frac{1}{2} \varepsilon_0 E^2 \]
  - Resistor: no energy stored, (all energy consumed)
# Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td>![Symbol]</td>
<td>$\Delta V = V_+ - V_- = \varepsilon$</td>
</tr>
<tr>
<td>Resistor</td>
<td>![Symbol]</td>
<td>$\Delta V = -IR$</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td>![Symbol]</td>
<td>$\Delta V = 0$ (→$R=0$, $L=0$, $C=0$)</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td>![Symbol]</td>
<td>$\Delta V = V_- - V_+ = -\frac{q}{C}$, $\frac{dq}{dt} = I$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>![Symbol]</td>
<td>$\Delta V = 0$ (→$R=0$, $L=0$, $C=0$)</td>
</tr>
<tr>
<td>Inductor</td>
<td>![Symbol]</td>
<td>$\Delta V = -L\frac{dI}{dt}$</td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td>![Symbol]</td>
<td>$L=0$, $C=0$, $R=0$ (on), $R=\infty$ (off)</td>
</tr>
<tr>
<td>Transformer</td>
<td>![Symbol]</td>
<td>Future Topics</td>
</tr>
</tbody>
</table>

Diodes, Transistors,...
LR Circuit

- Current as a function of time after switching on: $I(t)$

$$I(t) = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{L/R}}\right)$$

Note: the time constant is $\tau = L/R$

Quiz: What is the current when $t = \infty$?

Homework: “Switching off”
Turn on LR Circuit: Algebra Details

Apply Kirchhoff loop rule

\[ V_0 - IR - L \frac{dI}{dt} = 0 \]

\[ V_0 \, dt - IR \, dt - L \, dI = 0 \]

\[ \frac{dI}{(V_0 - IR)} = \frac{dt}{L} \]

\[ \int_0^I \frac{dI}{(V_0 - IR)} = \int_0^t \frac{dt}{L} \]

\[ -\frac{1}{R} \ln \left( \frac{V_0 - IR}{V_0} \right) = \frac{t}{L} \]

\[ I = \frac{V_0}{R} \left( 1 - e^{-\frac{t}{L/R}} \right) \]
LR Circuit: Time Constant

Turning on:

\[ I = \frac{V_0}{R} \left(1 - e^{-\frac{t}{L/R}}\right) \]

Turning off:

\[ I = I_0 e^{-\frac{t}{L/R}} \]

Time Constant of the LR circuit: \( \tau = \frac{L}{R} \)