Today’s Topics

- Faraday’s Law (Ch 31)
- Review Emf
- Change of Magnetic Flux and Emf (\(\mathcal{E}\))
- Lenz’s Law and its applications
- Formulation of Faraday’s Law
- Motional emf (details next Tuesday)

- Expected Preview: Ch 31.1-3
Sources of $\mathbf{E}$ and $\mathbf{B}$ Fields: An overview

- **Sources for the electric field:**
  - Electric charges (Coulomb's Law, static)
    
    \( \rightarrow \) subjects of past several weeks
  - Change of $\mathbf{B}$ field (Faraday's Law, varying in time)
    
    \( \rightarrow \) Today

- **Sources for the magnetic field:**
  - Electric current (Biot-Savart Law/Ampere’s Law, static)
    
    \( \rightarrow \) Last week
  - Change of $\mathbf{E}$ field (Ampere-Maxwell Law)
    
    \( \rightarrow \) Ch 34

\( \rightarrow \) All these features are summarized in Maxwell’s Equations.

\( \rightarrow \) Ch 34
Review: Electromotive Force (emf, $\varepsilon$)

- Electromotive “force”, emf, is a measure of the voltage that can be provided by a source.
  - emf is not a force, it has a unit of volts
  - sources of emf:
    - chemical process (battery)
    - change of magnetic flux
    - solar panel.....

- e.g. battery:
  - notice that emf has a direction
  - emf may exist even if no current.

$\varepsilon = 1.5V$
Demo: Emf and Change of Magnetic Flux

Also see: battery less flash light
Faraday’s Law of Induction

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit”.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Notes:
  - “Circuit”: any closed path, does not have to be a real conducting circuit
  - The path/circuit does not have to be circular, or even planar

\[ \Phi_B = \int B \cdot dA \]
Review: Flux

- Flux is definable with a vector field \( V \) and a surface
  \[
  \Delta \Phi_V = V \cdot \Delta A = V \Delta A \cos \theta \quad (V \text{ can be } E \text{ or } B)
  \]

e.g. Electric Flux through a closed surface
(Explained in Chapter 24)
Direction of Induced emf

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- \( \mathcal{E} > 0 \), same as nominal direction
- \( \mathcal{E} < 0 \), opposite

- Note that the nominal direction of \( \mathcal{E} \) and the direction of vector \( A \) follows right hand rule

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]
Lenz’s Law: Direction of emf

- The emf due to change of magnetic flux tends to create a current which produces a magnetic field to compensate the change of original magnetic flux.
  - Note: Real current may or may not be generated.
  - Lenz’s law is a convenient way to determine the direction of the emf due to magnetic flux change.
Lenz’s Law Explained

- Lenz’s law in plain words: the induced emf always tends to work against the original cause of flux change.

<table>
<thead>
<tr>
<th>Cause of $d\Phi_B/dt$</th>
<th>“Current” due to Induced $\varepsilon$ will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing $B$</td>
<td>generate $B$ in opposite dir.</td>
</tr>
<tr>
<td>Decreasing $B$</td>
<td>generate $B$ in same dir.</td>
</tr>
<tr>
<td>Relative motion</td>
<td>subject to a force in opposite direction of relative motions</td>
</tr>
</tbody>
</table>

Note: “Current” may not actually produced if no circuit
Exercises: Determine Direction Of emf Using Lenz’s Law

- Indicate the direction of emf in the following cases:

  - |B| increases
  - |B| decreases
  - |B| decreases
  - |B| increases
  - |B| decreases
  - path outside B

- or
Demo: Jumping Ring and more

switch on and
the ring jumps
Examples of Lenz’s Law

(a) 

(b) $I$ increasing

(c) $I$ decreasing
Back to Faraday’s Law of Induction

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit”.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Notes:
  - “Circuit”: any closed path → does not have to be real conducting circuit
  - The path/circuit does not have to be circular, or even planar

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]
Methods to Change Electric Flux

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \quad uniform \quad B \quad = \quad - \frac{d(BA \cos \theta)}{dt} \]

- Change of \( \Phi_B \rightarrow \text{emf} \)

  - To change \( \Phi_B \):
    - Change \( B \rightarrow \text{emf produced by an induced E field} \)
    - Change \( A \rightarrow \text{motional emf} \)
    - Change \( \theta \rightarrow \text{motional emf} \)
    - Combination of above
Demo: Electric Generator

\[ \mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{d(AB \cos \theta)}{dt} = NAB \omega \sin(\omega t) \]

\[ \theta = \omega t \]
Motional emf

- If a conducting segment are in relative motion (cut through) with a magnetic field, an emf is produced.
  - Motional emf is produced by the magnetic force on the free particles inside the conductor
  - Faraday’s law is also valid for this type of emf

Exercise: show that the motional emf in the left fig. is $\varepsilon = Blv$ (Details next Tuesday)