Physics 202, Lecture 12

Today's Topics

- Magnetic Field (Ch 29, part II)
- Force Between Two Current Carrying Wires
- Torque on a Current Loop in Uniform \( B \) Field
- Magnetic dipoles
- Motion of a Charged Particle In a Uniform \( B \) Field
- Applications:
  - Magnetic Confinement
  - First Modern Particle Accelerator: Cyclotron
  - Mass Selector (\( q/m \))
  - Hall Effect.

Magnetic Force On Current Carrying Wire Segment (Review of Last Lecture)

- Magnetic force on a current segment of length \( L \) in uniform field \( B \):
  \[ F_B = \sum qv \times B = IL \times B \]
- Key steps to derive:
  - Current: moving charges. \( I = qnA \)
  - Magnetic force on charge \( q \): \( qv \times B \)
  - \( F_B = qv \times B \) (ALn) = IL \times B

Magnetic Force On A Current Carrying Wire

- Magnetic force on a curved wire in uniform field \( B \):
  \[ F_B = IL \times B \]
- \( L' \) is the displacement perpendicular to the \( B \) field.

Note: Net force on a current loop in uniform \( B \) field is zero.

Also demo: Bending electron beam
Review Exercise: Forces On A Current Loop

For a current loop in a uniform magnetic field as shown, what is the direction of the force on each side?

Remember: \( F = I \times B \)

Note: \( \sum F = 0 \), ...

Case 1

Case 2

Torque on a Current Loop In Uniform \( B \) Field

Exercise: For a current loop in a uniform \( B \) field, show that the torque on the loop is: \( \tau = I A B \sin \theta \)

\( \mu \): magnetic dipole moment

\( I \): Area Vector

Review: Electric Dipole Moments

Electric dipole moment \( p \).

\( \sum F = 0 \)

\( \tau = p \times E \)

\( U = -p \cdot E \)

Magnetic Dipole Moments

Magnetic dipole moment \( \mu \).

\( \sum F = 0 \)

\( \tau = \mu \times B \)

\( \mu \): angular momentum of orbiting or spin

\( \mu \): in \( B \) Field
Quick Quiz 1

A magnetic dipole moment initially points at 45°. When a uniform horizontal B field is applied, which of the followings will happen?

1. No change B
2. Points towards the B field B
3. Points against the B field B
4. Points normal to the B field B

Quick Quiz 2

A magnetic dipole moment initially points at 135°. When a uniform horizontal B field is applied, which of the followings will happen?

1. No change B
2. Points towards the B field B
3. Points against the B field B
4. Points normal to the B field B

Side Story (Chapter 8): Stable and Unstable Equilibrium

\[ F_B = 0 \]
\[ T_B = 0 \]
\[ U = -\mu B \text{ (low)} \]
\[ (d^2U/d\theta^2 > 0) \]

\[ F_B = 0 \]
\[ T_B = 0 \]
\[ U = +\mu B \text{ (high)} \]
\[ (d^2U/d\theta^2 < 0) \]

Stable Equilibrium

Unstable Equilibrium

The picture is also true for electric dipole moment E

Motion Of Charged Particle in a Uniform B Field

Show that if a charged particle q of mass m in a uniform B field has an initial velocity v in the plane perpendicular to B, its motion is a uniform circular motion in that plane with:
- radius \( r = mv/qB \)
- period:
  \[ T = 2\pi \sqrt{m/qB} \]

Note: \( T \) is independent of v
(recall: uniform circular motion)
Motion Of Charged Particle in a Uniform \( \mathbf{B} \) Field – General 3D Case

- On the plane perpendicular to \( \mathbf{B} \):
  - \( R = \frac{mv}{qB} \)
  - \( T = \frac{2\pi m}{qB} \)
- Parallel to \( \mathbf{B} \)
  - spacing between helix
  - \( d = v_r T = \frac{v_r 2\pi m}{qB} \)

Application: Speed/Mass Selector

- Explain how the selectors work (next week’s lab)

  **Velocity Selector**

  - Speed selected:
    - \( v = \frac{E}{B} \)

  **Mass Selector**

  - Mass selected:
    - \( m/q = \frac{rB_0}{v} = \frac{rB_0}{(E/B)} \)

Mass Selector: J. J. Thomson Apparatus (1897)

- This is your lab next week: measuring \( e/m \)

Application: Cyclotron (First Modern Particle Accelerator)

- Explain how a cyclotron works (see board)

First Cyclotron (1934)
Lawrence & Livinston
Application: The Hall Effect

- Explain how Hall Effect works

\[ \Delta V_H = \frac{IB}{nq} \]

Application: Magnetic Confinement

- Tokamak
- Magnetic Bottle
- Van Allen Belts

MST: Madison Symmetric Torus