These problems contain the ideas I will choose from in writing the first mid-term examination. This examination will cover chapters 1-5 of the text, ending at the idea of friction (which is included in the mid-term).

1. A person stands at a horizontal distance (x) from a mountain and measures the angle of elevation of the mountaintop above the horizontal as (θ). After walking a distance (d) closer to the mountain on level ground, the person finds the angle to be (φ). Find a general equation for the height (y) of the mountain in terms of (d, θ, and φ), neglecting the height of the person’s eyes above the ground.

Answer:

(1) y = x tan(θ)
(2) y = (x-d) tan(φ)
(3) Therefore: x = y/[tan(θ)] = {y/[tan(φ)]} + d
(4) Which implies: y [(1/tan(θ)) – (1/tan(φ))] = d
(5) Or: y = d/[(1/tan(θ)) – (1/tan(φ))]

2. Why is the following situation impossible in one-dimensional motion? Starting from rest, an object moves 50.0 meter in a straight line in 10.0 s. The acceleration is constant during the entire period, and the final speed is 8.00 m/s.

Answer:

(1) 50.0 meter = (1/2) (a) (10)² = 50 a ----> a = 1.0 m/s²
(2) 8.00 m/s = (10.0) a ----> a = 0.80 m/s²
These are inconsistent.

3. A man drops a rock into a well. The man hears the sound of the splash 24.0 s. after he releases the rock from rest. The speed of sound in air is 336 m/s. How far is the water in the well below the top of the well?

Answer:

(1) Total time = 24.0 s. This can be divided into T_R , the time the rock falls before it hits the water, and T_S , the time the sound wave from the splash travels up the well. Let (D) be the distance from well top to water. Then:
(2) D = (1/2) (9.80) (T_R )² = (336) (T_S ) and
(3) T_R + T_S = 24.0 s.
(4) Combining (2) & (3):
(T_R )² + 68.6 (T_R ) – 1646 = 0 ----> T_R = 18.8 s. ----> T_S = 5.2 s. ----> D = 1750 m.
4. Consider a function \( V(x,y,z) = 20x^2 + 9.8z \)
a) Determine the gradient of \( V \). Call this vector \( \mathbf{F} \).
b) Determine the curl of \( \mathbf{F} \), written as \( \nabla \times \mathbf{F} \).
c) What is the divergence of \( \mathbf{F} \), written as \( \nabla \cdot \mathbf{F} \)?

Answer:
(a) \( \mathbf{F} = (40x) \mathbf{x} + 9.8 \mathbf{z} \)
(b) Because the curl of the gradient is always zero, the curl of \( \mathbf{F} = 0 \).
(c) \( \nabla \cdot \mathbf{F} = 40 \)

5. A projectile is fired with an initial speed of 20.0 m/s at an angle of 30° above horizontal from a cliff that is 45.0 m. above a flat plain. The mass of the projectile is 7.0 kg.
(a) Determine the speed of the projectile when it returns to a height of 45.0 m. above the plain.
(b) What is the height above the plain of the apogee?
(c) What is the velocity of the projectile just before it hits the plain?
(d) What is the time the projectile is in the air?

Answer:
(a) We separate the horizontal and vertical components of velocity. For part (a), this separation is, however, not needed, since the speed is 20.0 m/s.
(b) The initial vertical speed is 10.0 m/s. This means that the time to apogee is \( t = \frac{10.0}{9.80} = 1.02 \) s. The height of the projectile above the cliff is:
\[
[10.0 \times 1.02] - \left[ \frac{1}{2} \times 9.80 \times (1.02)^2 \right] = 5.1 \text{ m. So the height above the plain is } 50.1 \text{ m.}
\]
(c) Now we need to separate the horizontal and vertical components of velocity.
Horizontal: 10.0 m/s constant.
Vertical:
\[
(V_F)^2 - (V_I)^2 = 2az = 2 \times (9.80) \times (45.0) = (V_F)^2 - (10.0)^2 \quad \Rightarrow \quad V_F = 31.3 \text{ m/s}
\]
This means that the velocity = (10.0 m/s) \( \mathbf{x} \) – (31.3 m/s) \( \mathbf{z} \), where \( \mathbf{x} \) and \( \mathbf{z} \) refer to the horizontal and vertical axes.
(d) From the apogee to the plain is a distance of 50.1 m, starting from rest and acted on by gravity, so the elapsed time is:
\[
(50.1 \text{ m}) = \left( \frac{1}{2} \right) \times (9.80) \times (t)^2 \quad \Rightarrow \quad t = 3.20 \text{ s., so the total time is } 4.22 \text{ s.}
\]
6. A (20.0 kg) weight is suspended from a ceiling with two wires. The wire to the left is at an angle of 30° above the horizontal, while the wire to the right is at an angle of 45° above the horizontal. Determine the tension in each wire.

Answer:
Let the tension in the left wire and right wire be denoted by \( T_L \) and \( T_R \). Denote the horizontal axis by \( x \) and the vertical axis by \( z \). Then:

\[
\begin{align*}
\text{x-direction:} & \quad - T_L \cos (30^\circ) + T_R \cos (45^\circ) = 0 \\
\text{z-direction:} & \quad + T_L \sin (30^\circ) + T_R \sin (45^\circ) = (20.0 \text{ kg}) (9.80)
\end{align*}
\]

These two equations have solutions:
\( T_L = 144 \text{ newton}, \ T_R = 176 \text{ newton} \).

7. Two objects are connected by a frictionless pulley. The \( M_1 = (10.0 \text{ kg}) \) mass hangs vertically. The \( M_2 = (15.0 \text{ kg}) \) mass hangs on an incline that has an angle (\( \theta \)) above the horizontal. The incline is also frictionless. The objects do not move. Determine (\( \theta \)).

Answer: Set up a free-body force diagram for each object.
For \( M_1 \):
Gravity is straight down and the tension (T) in the wire is straight up:
\( T = (10.0 \text{ kg}) (9.80) = 98.0 \text{ newton} \)

For \( M_2 \):
The component of gravity parallel to the incline surface is (g) (sin \( \theta \)). The wire tension (T) is parallel to the incline surface:
\( T = (15.0) (9.8) \sin (\theta) = 98.0 \quad \text{------> \sin (\theta) = 2/3 \quad \text{------> \theta = 41.8^\circ}} \)

8. Same problem as #7, except there is friction on the incline surface, with coefficient of friction \( \mu = 0.12 \). Determine the angle (\( \theta \)).

Answer: Again, set up a free-body force diagram for each object.
For \( M_1 \):
This is the same as #7. \( T = 98.0 \text{ newton} \)

For \( M_2 \):
The restoring force (n) = \( M_2 \) g cos(\( \theta \))
The friction force \( F_{FR} = \mu \) (n) and opposes motion. So there are TWO answers, depending on whether \( M_2 \) would go down or up the incline in the absence of friction.
(i) Otherwise down:
\[ + M_2 \text{ g} \sin (\theta) - T - F_{FR} = 0 = 147 \sin (\theta) - 98.0 - \left[(0.12) (147)\right] \cos (\theta) \]
This can be rewritten as:

\[ 147 \sin (\theta) = 98 + 17.6 \cos (\theta) \]

Square both sides and use \( \sin^2(\theta) + \cos^2(\theta) = 1 \):

\[(147)^2 [1 - \cos^2 (\theta)] = (98)^2 + [(2) (98) (17.6)] \cos (\theta) + (17.6)^2 \cos^2 (\theta) \]

Combining terms:

\[ 21919 \cos^2 (\theta) + 3450 \cos(\theta) - 12005 = 0, \text{ or:} \]

\[ \cos^2 (\theta) + (0.157) \cos(\theta) - (0.548) = 0, \text{ which has solution:} \]

\( \theta = 48.2^\circ \).

(ii) Otherwise up:

\[ + M_2 \ g \sin(\theta) - T + F_{FR} = 0 = 147 \sin (\theta) - 98.0 + [(0.12) (147)] \cos (\theta) \]

Which can be rewritten as: \( 147 \sin (\theta) = 98 - 17.6 \cos (\theta) \)

Square both sides and use \( \sin^2(\theta) + \cos^2(\theta) = 1 \):

\[(147)^2 [1 - \cos^2 (\theta)] = (98)^2 - [(2) (98) (17.6)] \cos (\theta) + (17.6)^2 \cos^2 (\theta) \]

Combining terms:

\[ \cos^2 (\theta) - (0.157) \cos(\theta) - (0.548) = 0, \text{ which has solution:} \]

\( \theta = 34.6^\circ \).