The class lectures will include most- but perhaps not all- of the points covered in class. I also include fully worked out problems (FWOP) as examples to help you.

**Units of length, mass, time**

* Length: meter (39.37 inches), abbreviation “m”. Also encountered in this class is centimeter (10^{-2} meter), abbreviation “cm” and millimeter (10^{-3} meter), abbreviation “mm”.
* Mass: Until 1905, when the work of Albert Einstein was published, “mass” referred to the amount of “stuff” making up an object, and mass was regarded as a constant of the object. The unit is kilogram (2.21 pounds), abbreviation “kg”. Also encountered in this class is gram (10^{-3} kg), abbreviation “g” in standard way of writing, although I will use the older “gm” to distinguish gram from the acceleration due to gravity near the Earth’s surface, which is also abbreviated “g”.
* Time: Again until 1905, when the work of Albert Einstein was published, time referred to a constant, and the unit used is second, abbreviation “s”.

**Building blocks of matter**

Until the early 1800s, people had little idea of what made up matter. In the 1800s, two forms of electrical charge, called positive and negative, were found. In the period of 1900-1940, electrons, protons and neutrons were discovered, along with the present model of atoms consisting of nuclei with protons and neutrons orbited by electrons. After 1940, people learned that protons and neutrons are made up of constituent particles called “quarks,” while electrons have no internal constituents.

**Dimensional analysis**

The idea is simple: if two sides of an equation are equal, the sides have the same units. Thus, to say that “5.0 kg = 4.5 s” makes no sense, because the units differ. Consequently, it is possible to determine the dimensions of some variables by knowing the dimensions on each side of an equation.

**Example 1.1**: For example, suppose I know that an object moves at a constant speed of 15.0 m/s. How do I connect the amount of time (t) it moves and the distance (L) it moves?

L = ?. Well, the right hand side must have units of length, that is, meter. The two variables I know about are the speed (v = 15.0 m/s) and the elapsed time (t, in seconds). How do these two variables produce units of length? Answer: (v) (t) has units of length-meter. No other combination of (v) and (t) has such units, so the right hand side must involve (v) (t). Does this mean that:

L = (v) (t) ?

In general, the answer is no. Why? Because there might be a constant of proportionality, which has no units, and thus we could not figure out what the constant is. In this particular example, though, L = vt is indeed the answer.
Example 1.2: Suppose I know that the word “acceleration” \(a\) has units of meter/second\(^2\) or m/s\(^2\). Suppose that \(a\) is constant for a time of 4.5 s. How far \(L\) will the object move due to this acceleration? Again, what is the relation among the variables \((L), (a)\) and \((t = 4.5 \text{ s})\). If \((L)\) is on the left side of the equation, what combination of \((a)\) and \((t)\) yields units of meter\- length on the right side?

Answer: \((a) (t)^2\) is the only combination.

So, then, the equation MUST be \(L = at^2\)? NO! Remember, there is a constant of proportionality we do not get from dimensional analysis. In fact, as we will learn, the constant for this equation is \((1/2)\):

\[L = \frac{1}{2} at^2\] is the correct answer.

Example 1.3: Suppose an object is rotating around a circle of radius \((R)\) at a constant speed \((v)\). How do I express the change in angle with time? In order to start on this problem, I have to remind myself of something—anything—that connects length (unit: meter) and angle (unit: no dimension). Well, one connection is that the circumference of a circle is \((2\pi)\) radians, and has a length of \((2\pi R)\) meter. So the change of angle and change of length are related: a change \((\theta)\) of angle around this circle corresponds to a change of length \((\theta R)\). We also know, from example one, that \(R = vt\). This means that \(v = R/t\). This is the change of length/time, or speed. What of the change in angle during the same time \((t)\)?

Uncertainty in measurements and significant figures

Uncertainty: Suppose you measure the acceleration due to gravity in your laboratory room five times and obtain the following numbers (in meter/second\(^2\) or m/s\(^2\)): 9.80, 9.76, 9.81, 9.85, 9.77. What is the “real” number and what is the uncertainty in that number?

“Real” number ----> This is code for the average. So we take the average:

\[\frac{9.80 + 9.76 + 9.81 + 9.85 + 9.77}{5} = 9.80\]

Notice!! I did not write the average with more knowledge than I wrote the individual measurements. Since in this example I knew the individual measurements to 0.01 m/s\(^2\) or better, this is what I wrote the average as. My calculator can give me 10-15 digits after the decimal place, but I did not write them all because my knowledge is not that good.

What about uncertainty? ----> Here we use the idea of standard deviation, which is a statistical measure of how well we know an answer (a number). If I have five measurements \((g_1, g_2, g_3, g_4, g_5)\) with an average \(g_{\text{AVG}}\), then the standard deviation is:

\[\frac{1}{5-1} [(g_1 - g_{\text{AVG}})^2 + (g_2 - g_{\text{AVG}})^2 + (g_3 - g_{\text{AVG}})^2 + (g_4 - g_{\text{AVG}})^2 + (g_5 - g_{\text{AVG}})^2]^{1/2}\]

Notice that (a) it does not matter whether a given measurement is above or below the average—this standard deviation calculates how close or how far away a measurement is from the average; (b) the standard deviation gets smaller as the number \((5)\) of measurements get bigger, (c) the units are the same as the measurements. In our case we get a standard deviation of 0.018, or 0.02. Thus, we would write the average of the measurements as \(9.80 \pm 0.02 \text{ m/s}^2\).

Significant figures: The idea here is how well we know a measurement. There are two ways we can explain our knowledge and limitations. One is a fraction (a percentage) of the measurement.
For example, suppose we say we know the acceleration due to gravity in the laboratory to within 1.0%. What is that? The nominal value is 9.80 \text{ m/s}^2. 1\% of this is 0.098 \text{ m/s}^2. So we would write:

\[ 9.80 \pm 0.098 \text{ m/s}^2. \]

Wait, wait! We only know the nominal answer to 0.01, so how can we know the uncertainty to 0.001? Mistake! Instead, we should write, rounding up 0.098 to 0.10:

\[ 9.80 \pm 0.10 \text{ m/s}^2. \]

Another way to express our limitation is to say it in terms of an absolute uncertainty. So, for instance, we know our equipment and know it can measure acceleration only to ± 0.03 \text{ m/s}^2. How would we know this? We would measure many times and realize we can never get a more certain answer than ± 0.03 \text{ m/s}^2. Given this, we would express our uncertainty in how well we know the acceleration as: 9.80 ± 0.03 \text{ m/s}^2.

An important point to remember is that there is always an uncertainty to any measurement. This means, for example, that if a person says that global warming will raise the Earth’s temperature by 5° F, they have failed to tell you something important: how well do they know this estimate? Is it 5° ± 5° F, in which case there might be no change (or, equally likely, a 10° F change - even worse!), or 5° ± 0.5° F, in which case it is virtually guaranteed there will be an important rise in temperature? Here is a simple example in which the uncertainty makes all the difference in deciding what type of problem we are facing.

So, with uncertainties, how do we add/subtract, or multiply/divide number that have uncertainties?

**Add/subtract**: The number of decimal places - not the number of significant figures - of the result should be the smallest number of any term in the sum or difference.

**Example 1.4**:

Add: (5.01) + (4.5) + (2.7132) = ?

If I use a calculator blindly, I get 12.2232. This is wrong—none of the constituents have six significant figures. (4.5) has the smallest number of decimal places - one- so the answer is: 12.2

Just 12.2. Not 12, even though (4.5) has only two significant figures. Not (12.22), even though (5.01) and (2.7132) have two or more decimal places.

**Multiply/divide**: The number of significant figures - not the number of decimal places - of the result is the smallest number of significant figures of any term in the product.

**Example 1.5**:

Multiply: (5.01), (4.5), and (2.7132). Notice right away: (4.5) has the smallest number of significant figures- two- so the product will have two significant figures. The result?

61. Just 61, two significant figures, even though all three constituents have at least one decimal place.

**Conversion of units**
This section of chapter one uses many words to say you may need to have a formula to change from one expression to another of the same quantity. For example, temperature in degrees Fahrenheit has 32°F for the freezing point of water, and 212°F for the boiling point of water. In degrees Centigrade, it is 0°C and 100°C. How to convert one unit to the other? Come to class prepared by knowing, and knowing how to explain, the conversion formula.

**Coordinate systems**

There are many coordinate systems used in science & engineering. They (almost) all have one commonality: the directions of the coordinates are perpendicular to each other. Why? Because this means that what happens in one direction cannot influence what happens in the others. Let me go over the main coordinate systems you will encounter in this class, and then illustrate the importance of having perpendicular coordinates.

**Cartesian:** This is a fancy term for ordinary, x,y, z coordinates.

Notice that the three directions are mutually perpendicular. Does it matter which direction we label by which letter? Not at all. Is there anything special about this way of arranging the coordinates? Yes. The three directions are a “right-hand” coordinate system, in which if I take my right hand and swing my fingers from x-axis to y-axis, my thumb points in the +z direction. Likewise from y-axis to z-axis, my thumb points in the +x direction, and from z-axis to x-axis, my thumb points in the +y direction. Is there a reason to use a “right-hand” rather than a “left-hand” coordinate system? No, it is arbitrary, but you will see, and use, only right-hand coordinate systems in this class. So the two qualities of this coordinate system that matter are that the coordinates are perpendicular and that the coordinate system is right-handed.

**Spherical:** These are the coordinates of a sphere.

The three coordinates are the radial distance \((r)\) from the center of the coordinate system, the polar angle \((\phi)\) with respect to the z-axis, and the azimuthal angle \((\theta)\) with respect to the x-axis. Notice that all three coordinates are perpendicular to each other, as with the Cartesian coordinate system.

To convert from spherical to Cartesian coordinates, we use:

- \(z = r \cos(\phi)\)
- \(x = r \sin(\phi) \cos(\theta)\)
- \(y = r \sin(\phi) \sin(\theta)\)
**Problem solving strategy**

There is little I can say on this that is not inane. This class, and physics generally, is an exercise in pattern recognition- noticing what are the key parts to a situation and what does not matter (or matters less) and can be ignored. I will spend a great deal of time going through examples and emphasizing how to recognize the essential parts, and thus how to get started, on various types of problems.

**FWOP**

1.1 An automobile carburetor makes small particles of the gasoline and mixes the particles with air to obtain rapid combustion. Suppose that 30.0 cm\(^3\) of gasoline yields (N) identical spherical droplets, each with a radius of 2.00 x 10\(^{-5}\) m. What is the total surface area of these droplets?

**How do I start?**

I ask how many droplets I have. To get the number of droplets, since they are identical, I divide the volume of gasoline by the volume per droplet. What is the volume of one droplet? The droplet is a sphere, which has volume:

\[
(\frac{4}{3}) \pi R^3 = (\frac{4}{3}) \pi (2.00 \times 10^{-5} \text{ m.})^3 = 3.35 \times 10^{-14} \text{ m}^3.
\]

The volume of gasoline is 30.0 cm\(^3\) = 3.00 x 10\(^{-5}\) m\(^3\). This means we have N = (3.00 x 10\(^{-5}\)) / (3.35 x 10\(^{-14}\)) = 8.96 x 10\(^8\) droplets.

Each droplet has a surface area (remember- again- a sphere):

\[
4\pi R^2 = 4\pi (2.00 \times 10^{-5} \text{ m})^2 = 5.02 \times 10^{-9} \text{ m}^2,
\]

so the total surface area (A) is:

\[
A = (8.96 \times 10^8) (5.02 \times 10^{-9}) = 4.50 \text{ m}^2.
\]

1.2 An auditorium measures 40.0 m x 20.0 m x 12.0 m (+ 0.1 for each number), with air density of 1.20 ± 0.01 kg/m\(^3\). Determine (a) the volume of the room, and (b) the mass of air in the room, both with correct uncertainty.

(a) Volume is easy if the auditorium is a solid rectangle: \(V = (40.0) (20.0) (12.0) = 9.60 \times 10^3 \text{ m}^3\). What is the uncertainty? Each number has three significant figures, so the volume also has three significant figures: \(V = (9.60 \pm 0.01) \times 10^3 \text{ m}^3\).

(b) The mass of the air = volume x density = (1.15 ± 0.01) x 10\(^3\) kg.

1.3 You sell natural gas. Your employee sends you a formula for the consumption of natural gas of a customer. The volume (V) in millions of cubic feet and the time (t) in months are related by:

\[V = 1.50 t + (0.00800) t^2\]
To fit this information into your overall data base, you need the volume in terms of cubic feet and the time in terms of seconds. What equation relates these two?

Volume = \((10^6) V\)

time = ? How many seconds in one month?
time in seconds = \((365/12) (24)(3600) = 2.63 \times 10^6\) seconds in one month.

Place these into the equation ---> the new equation, for volume (in cubic feet) in terms of time (in seconds) is:
\((10^6 ) V = [1.50 (2.63 \times 10^6 ) ] t + [(8.00 \times 10^{-3} ) (2.63 \times 10^6 )^2 ] t^2 \)

Or \(V = 3.95 t + (5.53 \times 10^4 ) t^2\)

**Vectors and properties**

A vector is any object having both a size and a direction. In our class we concentrate on physical, three-dimensional spatial directions. If I tell you that I am walking at 3.5 miles/hour, this is not a vector—it has no direction. If I tell you I am walking due north, this is not a vector—it has no size. To be a vector, I must say that I am walking due north at 3.5 miles/hour.

Vectors come in all sorts of units. Here, we concentrate on displacement (location), with unit of meter, velocity (meter/second) and acceleration (meter/second^2). Later, we will discuss other vectors, including linear momentum (kilogram meter/second), force (kilogram meter/second^2), angular momentum (kilogram meter^2/second), and torque (kilogram meter^2/second^2). What all these have in common are size and direction- these are the only properties needed for an object to be a vector.

Like numbers, vectors can be added and subtracted. It is also possible to change the size of a vector by multiplying the vector by a number. Let’s go through these properties. First, though, two warning:

**Adding or subtracting two vectors can only be done if the vectors have the same units. So adding a displacement vector and a velocity vector makes no sense, because the result would have no single set of units.**

**Vectors have a size, but do not start at a particular place in the coordinate system. You will see many examples where vectors such as \(A = 3x + 5y\) start at \((0,0)\) in a two dimensional coordinate system and winds up at \((3,5)\). This is true, but not the whole truth. A vector that starts at \((1,1)\) and ends at \((4,6)\), or one that starts at \((-3,-5)\) and ends at \((0,0)\)- all these and many more are vector \(A\). This is because vectors tell you the change of location, both size and direction, but not the starting location in the coordinate system.**

Example 1.7: So, adding and subtracting vectors. Consider vectors \(A = 3x + 5y\) and \(B = -x + 2y + 4z\). What is:
\[ A + B = ? \]. Add the components in each direction, using the fact that the coordinates are perpendicular and components in one direction cannot affect the other directions.
\[ A + B = 2x + 7y + 4z \] (notice that \( A \) has no component in the \( z \)-direction, so \( 0 + 4 = 4 \))

\[ A - B = ? \]. Reverse the sign of each component of \( B \) and add to \( A \).
\[ A - B = 4x + 3y - 4z \].

\[ 2A + (1.5)B = ? \]. Multiply each vector by the number, then add the vectors, component to component.
\[ 2A + (1.5)B = 4.5x + 13y + 6z \]

\[ 2A + (N)B \] has no \( x \)-component. What is \( (N) = ? \). Since the problem calls out the \( x \)-component, this is the only component we need to look at. For the \( x \)-component, we have:
\[
[(2) (3)] + [(N) (-1)] = 0
\]
Or: \( N = 6 \) is the answer.

**Components of a vector**

There are only two important points here. If you use a coordinate system in which the coordinates are perpendicular, the components of a vector are independent of each other. Also, the projection of a vector onto any coordinate establishes a right triangle system and this allows us to determine the angle(s) between the vector and any coordinate direction. Let’s do a couple of examples.

**Example 1.8:**
Consider two vectors. \( A = 3x + 4y \) and \( B = 3x - 4y \). What angles do these vectors have with respect to the \( x \)-axis?
I chose these to illustrate two points: (i) both vectors have the same size = \( [3^2 + 4^2]^{1/2} = 5 \) units, and (ii) the projection of both vectors onto the \( x \)-axis is the same. Thus the enclosed angle between the \( x \)-axis and the vectors has the same magnitude. Notice!! Same magnitude, but opposite sign: the angle between the \( x \)-axis and \( A \) is \(+53^\circ\) , while that between the \( x \)-axis and \( B \) is \(-53^\circ\). Here the sign (+) or (-) uses the convention that counterclockwise from the coordinate axis is positive.

**Example 1.9:**
Suppose I know that vector \( A \) is in the \( xy \)-plane. I know the size of \( A \) is \((3.0)\) units. The projection of \( A \) along the \( x \)-axis, \( A_x = +2.1 \) units. What is the angle between vector \( A \) and the \( y \)-axis?
The first thing to note is that there may appear to be two answers. The vector \( A \) can have a positive or negative \( y \)-component, and one might think that this difference might change the angle between vector \( A \) and the \( y \)-axis. In fact, there are two answers, one positive and the other negative. The angle (\( \theta \)) between vector \( A \) and the \( x \)-axis is based on:

\[ \cos(\theta) = 2.1/3.0 = 0.70 \quad \Rightarrow \quad \theta = 45.6^\circ \]
since the angle between the vector and the y-axis is the complement of this, the answer is: 
\[ \pm 44.4^\circ \].

**Vector dot product**

Notice in example 1.8 we are projecting the vectors onto another direction, in this case the x-axis. There is a way to express, in general, the projection of one vector onto another: the vector dot product. The vector dot product is a number. It is defined two seemingly different, but equivalent, ways. Let’s use \( A_x \), \( A_y \), and \( A_z \) to denote the components of vector \( A \) in the x-, y-, or z-directions. Then the vector dot product is:

\[
A \cdot B = |A| |B| \cos(\theta) \quad \text{(Eq. 1.1)}
\]

where \( \theta \) is the included angle between the vectors. Notice that since \( \cos(\theta) = \cos(-\theta) \), it does not matter whether you refer to clockwise or counterclockwise rotation. The vector dot product can also be defined in terms of the components of the two vectors:

\[
A \cdot B = (A_x B_x) + (A_y B_y) + (A_z B_z) \quad \text{(Eq. 1.2)}
\]

These two equations are equivalent. What are the main uses of a vector dot product? One is certainly to find out how much of a given vector is in the direction of, and thus might influence, another vector. Let’s use this idea in a few examples.

**Example 1.10**: There is an object with a constant velocity vector \( \mathbf{v} = (2 \text{m/s}) \mathbf{x} - (5 \text{ m/s}) \mathbf{y} \). Suddenly, an acceleration vector \( \mathbf{a} = (4.9 \text{ m/s}^2) \mathbf{x} + (2.7 \text{ m/s}^2) \mathbf{y} \) is applied for 6.0 seconds.

(a) How much of \( \mathbf{a} \) changes the x-component of the velocity vector?

Answer: I want \( a_x \) or, equivalently \( (\mathbf{a} \cdot \mathbf{x}) \). Either way I do it, the answer is 4.9 m/s^2.

(b) What is the final velocity vector?

Answer: I make use of the fact that \( a_x \) only affects the x-component of velocity, and \( a_y \) only the y-component of velocity. So each component is a one-dimensional problem, for which we can apply:

\[
v_{xF} - v_{xI} = (a_x) \text{ (elapsed time)} = (4.9) \times (6.0) = 29 \text{ m/s}, \text{ so the final x-component of velocity is } 31 \text{ m/s}. \text{ Similarly:}
\]

\[
v_{yF} - v_{yI} = (a_y) \text{ (elapsed time)} = (2.7) \times (6.0) = 16 \text{ m/s}, \text{ so the final y-component of velocity is } 11 \text{ m/s}.
\]

**Example 1.11**: Suppose a (5.00 kg) object is moving at a constant velocity \( \mathbf{v} = (6.5 \text{ m/s}) \mathbf{x} \) on a flat, frictionless surface. A constant acceleration \( \mathbf{a} \) of magnitude 4.0 m/s^2 is applied at an angle of 30° above the horizontal. After 8.0 seconds, how fast is the object moving?

Answer: Only the horizontal part- the x-component- of \( \mathbf{a} \) contributes to changing \( \mathbf{v} \). What is this component?

\[
a_x = \mathbf{a} \cdot \mathbf{x} = a \cos(30^\circ) = (4.0) (\sqrt{3})/2
\]
**Vector cross product**

In dealing with torque and angular momentum later in the class, we will need the idea of a vector cross product, which is a vector defined by the following relation:

For vectors \( \mathbf{A} \) and \( \mathbf{B} \), the vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \) is defined by the components

\[
C_x = (A_y B_z) - (A_z B_y); \quad C_y = (A_z B_x - A_x B_z); \quad C_z = (A_x B_y - A_y B_x) \quad \text{(Eq. 1.3)}
\]

The vector \( \mathbf{C} \) is perpendicular to both \( \mathbf{A} \) and \( \mathbf{B} \) and the system of vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \) follow the right-hand rule, which says that if I use my right hand index finger straight out for the direction of \( \mathbf{A} \) and my middle finger at 90° with respect to the index finger for the direction of \( \mathbf{B} \), and stick my thumb perpendicular to both fingers, my thumb points in the direction of \( \mathbf{C} \).

Another aspect is the size of the vector cross product, of \( \mathbf{C} \):

\[
|\mathbf{C}| = |\mathbf{A}| |\mathbf{B}| \sin (\theta) \quad \text{(Eq. 1.4)}
\]

Where \( (\theta) \) is the magnitude of the angle between \( \mathbf{A} \) and \( \mathbf{B} \). A couple of important points follow from this. First, if the vectors \( \mathbf{A} \) and \( \mathbf{B} \) are parallel or antiparallel, then the vector cross product is zero because the included angle is zero. Also, the maximum size of the vector cross product occurs when the two vectors are perpendicular, since then the included angle is 90°. We will use this point when calculating the maximum torque or angular momentum. Let’s use the vector cross product idea in some examples.

**Example 1.12:** Suppose that vector \( \mathbf{A} = 3\mathbf{x} \), and the vector cross product is entirely in the –\( \mathbf{z} \) direction. What can we say, and not say, about the vector \( \mathbf{B} \)?

* First, since to get a vector cross product with a component in the –\( \mathbf{z} \) direction, we must have a component of \( \mathbf{B} \) in the –\( \mathbf{y} \) direction. Check the rules of components and make sure you understand why. Notice that we do not know the size of the component (from available information), merely that there must be such a component;

* Second, since the vector cross product is “…entirely in the –\( \mathbf{z} \) direction…”, vector \( \mathbf{B} \) cannot have any component in the ± \( \mathbf{z} \) direction. How come? Because such a component, in the vector cross product with \( \mathbf{A} \), yields a component in the ± \( \mathbf{y} \) direction, in contradiction to the statement of the problem;

* Lastly, there is something we cannot say from the information available: whether \( \mathbf{B} \) has any component in the ± \( \mathbf{x} \) directions. Why not? Because the vector cross product of \( \mathbf{A} \) and any such component of \( \mathbf{B} \) would be zero, so there is no way for us to know.

**Example 1.13:** Suppose vector \( \mathbf{A} = 2\mathbf{x} - \mathbf{y} + 5\mathbf{z} \) and vector \( \mathbf{B} = 4\mathbf{x} + \mathbf{y} - 3\mathbf{z} \). What is the vector cross product?

According to equations 3.3:

\[
C_x = [(1) (3)] - [(5) (1)] = -2
\]
C_y = [(5) (4)] - [(2) (-3)] = 26
C_z = [(2) (1)] - [(-1) (4)] = 6

In addition to these vector properties, there are some vector equations that require calculus. These include:

**Gradient:** A gradient is a vector. It is the derivative of a function. Suppose you have a function \( H(x,y) \) that tells you the height of the land with respect to sea level at different locations \((x,y)\). The gradient of \( H(x,y) \) is a vector pointing in the direction of the maximum change of \( H \).

\[
\nabla H = \left( \frac{\partial H}{\partial x} \right)_x x + \left( \frac{\partial H}{\partial y} \right)_y y
\]

(Eq. 1.5)

Here \( \left( \frac{\partial H}{\partial x} \right)_x \) means the derivative of \( H(x,y) \) with respect to \( x \) while holding \( y \) constant; this is called the “partial derivative” of \( H \) with respect to \( x \). Notice two points. (1) The vector points in the direction of the maximum increase of \( H(x,y) \), so \( -\nabla H \) would point in the direction of the maximum decrease of \( H(x,y) \); (2) The gradient of \( H(x,y) \) is an ‘ordinary’ vector, in having a size and a direction, so in these respects it is no different from any other vector.

**Curl of a vector:** Suppose \( F(x,y,z) \) is a vector. For example, \( F(x,y,z) \) might denote the size and direction of fluid flow at different locations within the fluid. Fluids can rotate—think whirlpools, for instance. How do we determine the tendency of a fluid to rotate about any given location \((x,y,z)\)? We use the curl of the vector:

\[
\nabla \times F = \left[ \left( \frac{\partial F_z}{\partial y} \right) - \left( \frac{\partial F_y}{\partial z} \right) \right] x + \left[ \left( \frac{\partial F_x}{\partial z} \right) - \left( \frac{\partial F_z}{\partial x} \right) \right] y + \left[ \left( \frac{\partial F_y}{\partial x} \right) - \left( \frac{\partial F_x}{\partial y} \right) \right] z
\]

(Eq. 1.6)

This vector points in the direction that the vector \( F \) rotates.

**Divergence:** Again, suppose \( F(x,y,z) \) is a vector. Suppose further that there is a source (or sink) that changes the size or direction of \( F \). For example, the sources of many electric fields are static (not moving) electrical charges. If I want to measure the size of a source, or sink, of a vector at a given point, how do I make such a measure? I use the divergence of the vector:

\[
\nabla \cdot F = \left( \frac{\partial F_x}{\partial x} \right) + \left( \frac{\partial F_y}{\partial y} \right) + \left( \frac{\partial F_z}{\partial z} \right)
\]

(Eq. 1.7)

For example, you will learn in physics 202 that as far as we have experimented there are no magnetic monopoles. Mathematically, we say this by saying that the divergence of the magnetic field \( B \) is zero:

\[
\nabla \cdot B = 0
\]

(Eq. 1.8)

This equation says there are no monopole sources or sinks of magnetic field.

**Mathematics Review & Summary**

**Algebra:**
The most important part of algebra for this class is solving algebraic equations, especially the quadratic equation:

\[ ax^2 + bx + c = 0. \]

To see how the solutions are found, note the following:

\[ ax^2 + bx + c = 0 \text{ can be modified to: } x^2 + (b/a) x + (c/a) = 0 \]

which can be modified to:

\[ (x + b/2a)^2 + (c/a) = (b/2a)^2 \]

which can be rewritten as:

\[ (x + b/2a)^2 = (b/2a)^2 - (c/a) \]

taking the square root of both sides:

\[ x + b/2a = \pm \{(b/2a)^2 - (c/a)\}^{1/2} \]

which can be rewritten as:

\[ x = -(b/2a) \pm \{(b/2a)^2 - (c/a)\}^{1/2} \]

another way to write this is:

\[ x = (1/2a) [-b \pm \{b^2 - 4ac\}^{1/2}] \]

**Example 1.6:** \( x^2 + 3x - 2 = 0 \). Solutions?

\[ x = (1/2) [-3 \pm \{3^2 - [4(1)(-2)]\}^{1/2}] = (1/2) [-3 \pm \{17\}^{1/2}] = +0.56 \text{ or } -3.56 \]

**Trigonometry:**

We use a right angle triangle. We start with the Pythagorean theorem:

\[ c^2 = a^2 + b^2 \]

Basic trigonometric functions are defined using the angle \( \theta \) at the vertex of the AC and AB lines:

\[
\begin{align*}
\sin \theta &= \sin(\theta) = a/c \\
\cos \theta &= \cos(\theta) = b/c \\
\tan \theta &= \tan(\theta) = a/b
\end{align*}
\]

These functions have several properties worth knowing well:

* \( \sin(2\theta) = 2\sin(\theta)\cos(\theta) \)
* \( \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \)
* \( \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \)
* \( \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \)

* **Law of Sines:** Let \( \alpha, \beta, \text{ and } \gamma \) be the angles opposite to sides \( a, b \text{ and } c \). Then:

\[
a/\sin(\alpha) = b/\sin(\beta) = c/\sin(\gamma)
\]

This Law of Sines is correct for all triangles, not just for right angle triangle.
* Law of Cosines: Again defining the angles and sides as per the Law of Sines, the Law of
Cosines can be written in three equivalent forms:

\[ c^2 = a^2 + b^2 - 2ab \cos(\gamma) \]
\[ a^2 = b^2 + c^2 - 2bc \cos(\alpha) \]
\[ b^2 = a^2 + c^2 - 2ac \cos(\beta) \]

These equations also apply to all triangles, not merely right angle triangles.

Calculus
This summary is not comprehensive. Instead, I merely note a few points you need to know,
including the ideas of a derivative, an integral and a partial derivative.

Derivative
The derivative of a function \( f(x) \) with respect to the variable \( x \), written as \( df/dx \), is the change of
\( f \) with respect to \( x \). For examples:

* \( f(x) = 2x^2 + 4 \frac{x^{4/3}}{3} + 8 \) \( \quad \Rightarrow \frac{df}{dx} = 4x + (16/3) x^{1/3} \) . Note that the last term \( (8) \) in \( f(x) \) has a
zero derivative, since it is constant, and that the first two terms obey:
\[ f(x) = A x^b \quad \Rightarrow \frac{df}{dx} = (A x^b) x^{b-1}. \]

* \( f(x) = 5 e^{2x} \) \( \quad \Rightarrow \frac{df}{dx} = 10 e^{2x} \). This is based on the general point that \( d[e^{ax}] / dx = a e^{ax} \),
which occurs because of the definition of the exponential function:
\[ e^{ax} = 1/1! + (ax)/1! + (ax)^2/2! + (ax)^3/3! + (ax)^4/4! + \ldots \quad \text{[Recall that } n! = n(n-1)(n-2)\ldots(2)(1)\text{]} \]
Using the power law rule for derivatives, we get the result written above for \( f(x) = 5 e^{2x} \).

* \( f(x) = A \sin(kx) + B \cos(mx) \) \( \quad \Rightarrow \frac{df}{dx} = (Ak) \cos(kx) - (Bm) \sin(mx) \)
This arises from the definitions of sine of \( (kx) \), written \( \sin(kx) \), in terms of powers of \( (kx) \), and
cosine of \( (mx) \), written \( \cos(mx) \), in terms of powers of \( (mx) \).

Integral
There are two related, but distinct, types of integrals: definite and indefinite. A definite integral
of \( f(x) \) over a range of \( x \) from \( x = x_i \) to \( x = x_F \) is the net area under the curve \( f(x) \) over the range of
\( x \). An indefinite integral is the function \( G(x) \) whose derivative is \( f(x) \). So, for example, the
indefinite integral of \( f(x) = 3 x^3 \) is \( \quad \Rightarrow G(x) = (3/4) x^4 + C \), since \( dG/dx = f(x) \). Here \( C \) is
any constant number; it is part of \( G(x) \) because \( d(C)/dx = 0 \) and thus we only know \( G(x) \) to
within an arbitrary constant.

Partial derivative
These are derivatives of a function of more than one variable taken with respect to one variable,
with the other variables held constant. For example:
\[ F(x,y,z) = 3xy + 4y^2 + 8 xz^3 -6 \]
What are the partial derivatives of $F$ with respect to $(x)$, $(y)$, or $(z)$?

$\frac{\partial F}{\partial x} = 3y + 8z^3$

$\frac{\partial F}{\partial y} = 3x + 8y$

$\frac{\partial F}{\partial z} = 24 xz^2$.

If you have trouble getting these answers, work on it until you understand where they come from.