Theory Uncertainties in Higgs Searches Using Exclusive Jet Bins

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based on work with
Iain Stewart, Wouter Waalewijn, Carola Berger, Claudio Marcantonini
Outline

1. Introduction
2. Counting Jets at Fixed Order
3. Resummation at NNLL+NNLO
4. More Jets
5. Summary
Introduction

Counting Jets at Fixed Order

Resummation at NNLL+NNLO

More Jets

Summary
Exclusive Jet Measurements

We are often interested in differential (exclusive) jet measurements

- Backgrounds vary with the number of jets

⇒ Be exclusive in the number of jets
  - \( pp \rightarrow H(\rightarrow WW^*) + 0, 1, 2 \text{ jets} \)
  - Also relevant for \( H \rightarrow \gamma\gamma \)

⇒ The “spectrum” of jets is also important for theory uncertainties

Other ways of being exclusive

- Jet mass and shape (e.g. distinguish quark and gluon jets)
- Jet substructure, e.g. to search for \( H \rightarrow b\bar{b} \)
- Photon and lepton isolation cuts, e.g. \( H \rightarrow \gamma\gamma \)
$H \rightarrow WW$ vs. $tt \rightarrow WWbb$

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⇒ Veto events with central jets, measure $pp \rightarrow H(\rightarrow WW) + 0$ jets
How to Veto Central Jets

Conventional: Jet algorithm

- Search for jets and require $p^\text{jet}_T < p^\text{cut}_T$
  - Tevatron: $p^\text{cut}_T \simeq 20$ GeV
  - LHC: $p^\text{cut}_T \simeq 25$ GeV

⇒ Complicated phase-space restrictions
How to Veto Central Jets

Conventional: Jet algorithm

- Search for jets and require $p_T^{\text{jet}} < p_T^{\text{cut}}$
  - Tevatron: $p_T^{\text{cut}} \simeq 20 \text{ GeV}$
  - LHC: $p_T^{\text{cut}} \simeq 25 \text{ GeV}$

⇒ Complicated phase-space restrictions

Alternative: Event shape

- Measure “beam thrust” of each event

$$\mathcal{T}_{\text{cm}} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_{kz}|)$$

and require $\mathcal{T}_{\text{cm}} < \mathcal{T}^{\text{cut}}$

⇒ Better suited to analytic calculations
Large Logarithms from Jet Veto

Even if hard signal process \( gg \rightarrow H \) contains no jets, jet veto affects cross section by restricting ISR

\[ \Rightarrow \ t\text{-channel singularities produce Sudakov double logarithms} \]

\[
\sigma(p_T^{\text{cut}}) \propto 1 - \frac{3\alpha_s}{\pi} \ln^2 \frac{p_T^{\text{cut}}}{m_H} + \ldots
\]

\[
\sigma(T^{\text{cut}}) \propto 1 - \frac{3\alpha_s}{\pi} \ln^2 \frac{T^{\text{cut}}}{m_H} + \ldots
\]

- Large perturbative corrections at small cuts
- Are larger for \( p_T^{\text{cut}} \) than \( T^{\text{cut}} \), agree for \( T^{\text{cut}} \simeq m_H \left( \frac{p_T^{\text{cut}}}{m_H} \right)^{\sqrt{2}} \)
Perturbative Structure of Cross Section

\[ L^2 = 2 \ln^2 \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L^2 = \ln^2 \frac{T^{\text{cut}}}{m_H} \]

\[ \sigma(p_T^{\text{cut}}) = 1 + \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s n_1(p_T^{\text{cut}}) \quad \text{NLO} \]
\[ + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \alpha_s^2 n_2(p_T^{\text{cut}}) \]
\[ + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \cdots \]
\[ + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \cdots \]

Inclusive cross section: \( p_T^{\text{cut}} \sim m_H \) so \( L \ll 1 \)

- Dominated by nonlogarithmic terms \( \alpha_i^s + \alpha_i^s n_i(p_T^{\text{cut}}) \)

⇒ Use fixed-order expansion: LO, NLO, NNLO
Perturbative Structure of Cross Section

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\[ + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \cdots \]

\[ + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \cdots \]

Inclusive cross section: \( p_T^{\text{cut}} \sim m_H \) so \( L \ll 1 \)

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\[ \Rightarrow \quad \text{Use fixed-order expansion: LO, NLO, NNLO} \]
Perturbative Structure of Cross Section

\[ \sigma(p_T^{\text{cut}}) = 1 \]
\[ + \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s n_1(p_T^{\text{cut}}) \]
\[ + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \alpha_s^2 n_2(p_T^{\text{cut}}) \]
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\[ + \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \cdots \]

LL

Inclusive cross section: \( p_T^{\text{cut}} \sim m_H \) so \( L \ll 1 \)
- Dominated by nonlogarithmic terms \( \alpha_s^i + \alpha_s^i n_i(p_T^{\text{cut}}) \)
⇒ Use fixed-order expansion: LO, NLO, NNLO

0-jet cross section: \( p_T^{\text{cut}} \ll m_H \) so \( L \gg 1 \)
- Dominated by logarithmic terms \( \alpha_s^i L^j, n_i(p_T^{\text{cut}}) \sim O(p_T^{\text{cut}}/m_H) \)
- Use resummed perturbation theory: LL, NLL, NNLL, N^3LL
Perturbative Structure of Cross Section

\[ L^2 = 2 \ln^2 \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L^2 = \ln^2 \frac{T^{\text{cut}}}{m_H} \]

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\[ + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots + \cdots \quad \text{LL, NLL} \]

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- Use resummed perturbation theory: LL, NLL, NNLL, N^3LL
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\[ \sigma(p_T^{\text{cut}}) = 1 \]

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\[ + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \cdots \]

LL NLL NNLL

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$$+ \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \cdots$$

$$+ \vdots + \vdots + \vdots + \vdots + \vdots + \vdots + \cdots$$

LL, NLL, NNLL, N^3LL

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- Use resummed perturbation theory: LL, NLL, NNLL, N^3LL
Counting Jets at Fixed Order

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Counting Jets at Fixed Order

Fully differential NNLO known numerically
[Anastasiou, Melnikov, Petriello; Grazzini]
- FO expansion gets unstable at small $p_T^{\text{cut}}$ and eventually breaks down
- Naively, jet veto appears to improve convergence

Current recipe being used by experiments [Anastasiou et al., arXiv:0905.3529]
- Common scale variation for jet bins, e.g. for the Tevatron

$$\frac{\Delta \sigma}{\sigma} = 66.5\% \times \begin{pmatrix} +5\% \\ -9\% \end{pmatrix} + 28.6\% \times \begin{pmatrix} +24\% \\ -22\% \end{pmatrix} + 4.9\% \times \begin{pmatrix} +78\% \\ -41\% \end{pmatrix} = \begin{pmatrix} +14\% \\ -14\% \end{pmatrix}$$

0 jets 1 jet $\geq$ 2 jets

Smaller uncertainty in 0-jet bin than in inclusive cross section
Perturbative Structure of Jet Cross Sections

\[ \sigma_{\text{total}} = \int_0^{p_T^{\text{cut}}} d p_T \frac{d \sigma}{d p_T} + \int_{p_T^{\text{cut}}}^{\infty} d p_T \frac{d \sigma}{d p_T} \]

\[ \sigma_0(p_T^{\text{cut}}) + \sigma_{\geq 1}(p_T^{\text{cut}}) \]

\[ \sigma_{\text{total}} = 1 + \alpha_s + \alpha_s^2 + \cdots \]

\[ \sigma_{\geq 1}(p_T^{\text{cut}}) = \alpha_s(L^2 + L) + \alpha_s^2(L^4 + L^3 + L^2 + L) + \cdots \]

\[ \sigma_0(p_T^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}}) \]

\[ = [1 + \alpha_s + \alpha_s^2 + \cdots] - [\alpha_s(L^2 + L) + \alpha_s^2(L^4 + \cdots) + \cdots] \]

- Perturbative series in \( \sigma_{\text{total}} \) and \( \sigma_{\geq 1}(p_T^{\text{cut}}) \) have different structures and are unrelated
- Apparent small uncertainties in \( \sigma_0(p_T^{\text{cut}}) \) arise from cancellation between two series with large corrections
Division Into Jet Bins

To first approximation, one should treat perturbative series in $\sigma_{\text{total}}$, $\sigma_{\geq 1}$, $\sigma_{\geq 2}$ as independent with uncorrelated perturbative uncertainties, i.e.

1. Consider \textit{inclusive} jet cross sections

$$\sigma_{\text{total}}, \sigma_{\geq 1}, \sigma_{\geq 2} \Rightarrow C = \begin{pmatrix} \Delta_{\text{total}}^2 & 0 & 0 \\ 0 & \Delta_{\geq 1}^2 & 0 \\ 0 & 0 & \Delta_{\geq 2}^2 \end{pmatrix}$$

2. Transform to \textit{exclusive} jet cross sections

$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}, \quad \sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}, \quad \sigma_{\geq 2} \Rightarrow C = \begin{pmatrix} \Delta_{\text{total}}^2 + \Delta_{\geq 1}^2 & -\Delta_{\geq 1}^2 & 0 \\ -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 + \Delta_{\geq 2}^2 & -\Delta_{\geq 2}^2 \\ 0 & -\Delta_{\geq 1}^2 & \Delta_{\geq 2}^2 \end{pmatrix}$$
Realistic Fixed-Order Scale Uncertainties

- Uncertainties reproduce naive scale variation at large cut values
- Larger uncertainties at small cut values → take into account presence of large logarithmic corrections

\[ p_T^{\text{cut}} = 30 \text{ GeV} \]

<table>
<thead>
<tr>
<th>method</th>
<th>$\Delta \sigma_{\text{total}}$</th>
<th>$\Delta \sigma_0$</th>
<th>$\Delta \sigma_1$</th>
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<td>14%</td>
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<td>17%</td>
<td>29%</td>
<td>45%</td>
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Resummation at NNLL+NNLO
Resummation of Jet-Veto Logarithms

\[ L^2 = 2 \ln^2 \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L^2 = \ln^2 \frac{T_{\text{cut}}}{m_H} \]

\[ \sigma_{0\text{-jet}} = 1 + \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s n_1(p_T^{\text{cut}}) + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \alpha_s^2 n_2(p_T^{\text{cut}}) \quad \text{NLO} \]

\[ + \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \cdots \]

Initial-state parton shower resums LL

- Pythia/Herwig is LL (plus some NLL)
- MC@NLO, POWHEG: combine parton-shower LL with fixed NLO
Resummation of Jet-Veto Logarithms

\[ L^2 = 2 \ln^2 \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L^2 = \ln^2 \frac{T_{cm}^{\text{cut}}}{m_H} \]

\[ \sigma_{0\text{-jet}} = 1 \]

\[ + \ \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s n_1(p_T^{\text{cut}}) \]

\[ + \ \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \alpha_s^2 n_2(p_T^{\text{cut}}) \]

\[ + \ \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \ldots \]

\[ + \ : \ + \ : \ + \ : \ + \ : \ + \ : \ + \ : \ + \ldots \]

\[ \text{LL} \quad \text{NLL} \quad \text{NNLL} \quad \text{N}^3\text{LL} \]

Initial-state parton shower resums LL

- Pythia/Herwig is LL (plus some NLL)
- MC@NLO, POWHEG: combine parton-shower LL with fixed NLO

Our calculation: NNLL+NNLO

- Resummation using \( T_{cm}^{\text{cut}} \) and SCET \( \rightarrow \) two orders beyond PS
- \( n_{1,2}(T_{cm}^{\text{cut}}) \) numerically from FEHiP \( \rightarrow \) reproduces full NNLO
Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Pirjol, Stewart]

A general framework derived from QCD with which we can study what goes on before and after the hard interaction

**Soft**

Low-energy particles without preferred direction

**Collinear**

Energetic jets along incoming and outgoing directions
Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Pirjol, Stewart]

A general framework derived from QCD with which we can study what goes on before and after the hard interaction

**Soft**
Low-energy particles without preferred direction

**Collinear**
Energetic jets along incoming and outgoing directions

Advantages of SCET

- Clear separation of different contributions from different energy scales
  - Makes it easier to find the right probability formula for a given problem
  - Can expand in $\alpha_s$ when possible, work to all orders otherwise

- All required approximations are made explicit
  - Corrections can be computed systematically
Factorization for Exclusive Jet Cross Sections

Contributions appear at different physical energy scales $\Rightarrow$ Factorization

\[ d\sigma = \text{hard interaction} \otimes \text{PDFs} \otimes \text{ISR} \otimes \text{FSR} \otimes \text{soft radiation} \]

\[ d\sigma = H_N \times \left( (f_{a,b} \otimes I_{a,b}) \times \prod_{j=1}^{N} J_j \right) \otimes S_N \]

- SCET allows us to derive factorized cross section
  - Precise form depends on the specific observable
  - Each function has a precise definition in the effective field theory
Factorization Theorem for Beam Thrust

\[
\frac{d\sigma}{dT_{cm}} = H_{gg}(\mu) \int dt_a dt_b B_g(t_a, \mu) B_g(t_b, \mu) S_{gg}^B \left( T_{cm} - \frac{t_a + t_b}{m_H}, \mu \right)
\]
Factorization Theorem for Beam Thrust

\[
\frac{d\sigma}{dT_{\text{cm}}} = H_{gg}(\mu) \int dt_a dt_b \ B_g(t_a, \mu) \ B_g(t_b, \mu) \ S_{gg}^B \left( T_{\text{cm}} - \frac{t_a + t_b}{m_H}, \mu \right)
\]

Logarithms are split apart and resummed using RGE

\[
\ln^2 \frac{T_{\text{cm}}}{m_H} = 2 \ln^2 \frac{m_H}{\mu} - \ln^2 \frac{T_{\text{cm}} m_H}{\mu^2} + 2 \ln^2 \frac{T_{\text{cm}}}{\mu}
\]

\[\Rightarrow \mu_H \simeq m_H, \mu_B \simeq \sqrt{T_{\text{cm}} m_H}, \mu_S \simeq T_{\text{cm}}\]
Perturbative uncertainties can now be determined from independent scale variations

1. Overall scale by factor of 2
   - Equivalent to FO scale variation
   - Gives uncertainties at large cut

2. $\mu_B(T_{cm})$ profile
3. $\mu_S(T_{cm})$ profile
   - Dominate at small cut
   - Explicitly take into account uncertainties due to logarithms

$\Rightarrow$ Combine in quadrature $\max(\mu_H)$ and $\max(\mu_B, \mu_S)$ variations

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Convergence of Resummed Perturbation Theory

- Perturbative corrections are sizable
- Resummed perturbation theory shows good convergence with reliable uncertainties
**Comparison of Fixed Order and Resummation**

![Graph showing comparison of fixed order and resummation at NNLL+NNLO](image)

- **Resummation** is important to get the best central value and uncertainties.

- Scale uncertainty at NNLL+NNLO is 10–20\% → still larger than naive FO.

- It is feasible to further reduce uncertainties by going to N^{3}LL.

- Irreducible $pp \rightarrow WW$ background with the same cut can be computed using the same methods.
More Jets
N-Jettiness Event Shape


\[ \mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_{a}(p_k), d_{b}(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\} \]

- \( d_{a,b}(p_k), d_j(p_k) \): Distance of particle \( k \) to beam and jet directions
- Divides phase space into \( N \) jet regions and 2 beam regions
N-Jettiness Event Shape


\[ \mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\} \]

\[ \equiv \mathcal{T}^a_N + \mathcal{T}^b_N + \mathcal{T}^1_N + \cdots + \mathcal{T}^N_N \]

- \(d_{a,b}(p_k), d_j(p_k)\): Distance of particle \(k\) to beam and jet directions
- Divides phase space into \(N\) jet regions and 2 beam regions
- Can measure separate contribution from each region

For small \(\mathcal{T}^i_N\) final state contains exactly \(N\) jets

⇒ Enforcing small beam-thrust components \(\mathcal{T}^a_N + \mathcal{T}^b_N\) eliminates contamination from ISR
N-Jettiness with Geometric Measure

\[ \mathcal{T}_N = \sum_k |\vec{p}_k T| \min \{ d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k) \} \]

\[ \equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N \]

Geometric measure

\[ d_{\alpha,\beta}(p_k) = e^{\mp \eta_k} \]

\[ d_j(p_k) = 2 \cosh \Delta \eta_{jk} - 2 \cos \Delta \phi_{jk} \approx (\Delta \eta_{jk})^2 + (\Delta \phi_{jk})^2 \]

- Yields almost circular jets with size determined by \( \eta \)
- \( \mathcal{T}_N^j \) determines mass of jet: \( M_j^2 = |\vec{p}_T^j| \mathcal{T}_N^j \)
**Factorization for N-Jet Production**

- *Exclusive* N-jet cross section contains similar double logarithms from restricting ISR and FSR

⇒ Can be resummed using N-jettiness

Explicit factorization formula is known for N-jettiness

\[
\frac{d\sigma}{d\mathcal{T}^a_N \, d\mathcal{T}^b_N \, d\mathcal{T}^1_N \cdots d\mathcal{T}^N_N} = H_N \left[ B_a \times B_b \times \prod_{j=1}^{N} J_j \right] \otimes S_N
\]

⇒ All necessary ingredients for resummation at NNLL are available for any process for which virtual NLO diagrams are known such as \( H + 1, 2 \) jets
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\]

⇒ All necessary ingredients for resummation at NNLL are available for any process for which virtual NLO diagrams are known such as $H + 1, 2$ jets

More applications

- N-jettiness applied to jet substructure: N-subjettiness [Kim; Thaler, Tilburg]
  - As powerful as other methods to identify boosted heavy objects
  - Theoretically clean and factorizable
Summary and Outlook

One must be careful when applying fixed-order calculations and evaluating perturbative uncertainties:

- $H + 0 \text{ jets}$ cross section has typical behavior of a kinematic endpoint region involving large Sudakov logarithms.
- The same logarithms also appear for $H + \geq 1 \text{ jets}$.

$\Rightarrow$ The same applies to new-physics measurements using jets.

SCET provides a powerful formalism to analyze processes with jets:

- Can combine fixed order with a resummation of logarithms appearing in exclusive jet cross sections.
- Resummation provides more handles to reliably evaluate theory uncertainties.

Extension to N jets:

- N-jettiness provides a theoretically ideal exclusive N-jet algorithm.
- All ingredients are available for resummation at NNLL+NLO.
Physical Picture of Initial State

Measurement probes PDFs at some intermediate scale $\mu_B$

$\mu < \mu_B$: On-shell partons “inside” incoming proton
- ISR captured by PDF evolution, redistributes momentum fraction $x$

incoming beam

$\mu_{\Lambda}$ changing $x$ $\mu_B$ $\mu_H$

hard interaction
Physical Picture of Initial State

Measurement probes PDFs at some intermediate scale $\mu_B$

$\mu < \mu_B$: On-shell partons “inside” incoming proton
- ISR captured by PDF evolution, redistributes momentum fraction $x$

$\mu > \mu_B$: Off-shell parton ($-t < 0$) part of incoming jet
- Colliding parton emits collinear and soft ISR “outside” proton
- ISR goes into final state and is measured by jet veto
\( \pi^2 \) Resummation

- **Hard virtual corrections contain large \( \ln^2 (-1 - i0) = -\pi^2 \) terms**
  
  \[
  H_{gg}(m_H, \mu_H) \propto 1 - \frac{\alpha_s(\mu_H) C_A}{2\pi} \ln^2 \frac{-m_H^2 - i0}{\mu_H^2} + \ldots
  \]

  - Convergence improves significantly when \( \ln^2 (-1) \) are also resummed
  
  \( \Rightarrow \) Large K factors mostly from hard virtual corrections
Reproducing Fixed Order Result at Large $T_{cm}$

At large $T_{cm}$ (no jet veto)

- Exactly reproduce central value and uncertainties of fixed NNLO (using $\mu_{FO} = m_H$)

**NNLL+NNLO with $\pi^2$ summation (default) vs. fixed NNLO (using $\mu_{FO} = m_H/2$)**

- Central values agree at large $T_{cm}$
- $\pi^2$ summation also reduces scale uncertainty in total cross section
  - $+3\%, -5\%$ at LHC
  - $+5\%, -9\%$ at Tevatron