Energy received/s on A at in dσ (I received)

\[ I = I_{\text{emitted}} \cdot \Omega' \cdot A = I_0 \cdot \Omega' \cdot A \]

↑ (photons emitted from outside)
A will not be in cone dσ even if they strike
A (R)

Energy received/s on A at in dσ is:

\[ I_{\text{emitted}} (\theta) \cdot \Omega' \cdot A' \]

\[ = I_0 \cos \theta \cdot \Omega' \cdot \frac{A}{\cos \theta} \text{ is indep. of } \theta. \]
Disk at distance $R$ within slab emits $\frac{p \, dV}{s}$ ergs/s = $p \, r^2 \, d\Omega \, dr$ ergs into $4\pi \, sr$.
Fraction of this that hits $dA$ is $\frac{dA}{4\pi \, r^2}$.

:. Energy striking $dA$ in $d\Omega$ /s:

$$I(\theta) \, dA \, d\Omega = \int_{r = \text{near edge of slab}}^{r = \text{far edge of slab}} \frac{dA}{4\pi \, r^2} \, p \, r^2 \, d\Omega \, dr$$

Let $x = r \cos \theta$, $dx = dr \cos \theta$.

$$I(\theta) \, dA \, d\Omega = \int_{x = D + d}^{x = D} p \, \frac{dA \, dx}{4\pi \, \cos \theta} = \frac{p \, dl}{4\pi \, \cos \theta} \, dA \, dl$$

Or $I(\theta) = \frac{p \, dl}{4\pi \, \cos \theta}$. Note that the path length $dl$ through the slab along the line of sight is just $l = \frac{d}{\cos \theta}$, and that the observed intensity is independent of $D$ and depends only on $p$. ($I(\theta) = \frac{pl}{4\pi}$)
3.

\[ U = \text{ergs/cm}^3 \]

\[ \text{d}V = \text{volume between } r \text{ and } r + dr \text{ in cone} = r^2 \text{d}r \text{d}A \text{d}r. \text{ Total energy of photon in} \]

This volume = \( U \text{d}V \). These photons go

in random direction over \( 4\pi \). \text{Fraction that will hit hole is} \( \frac{A\text{hole}}{4\pi} \).

\[ A\text{hole} = \frac{dA \cos \theta}{\pi} \]

Total that will emerge in time \( \text{dt} \) is

evaporating with \( C \text{dt} \) of the hole. Then it is going in the right direction:

\[ \text{Power leaving} = I_{\text{exit}} \theta \text{d}A \text{d}A \text{d}t \]

\[ = \int_{r=0}^{r=\text{cutoff}} \frac{U \text{d}V}{4\pi} = \int_{r=0}^{r=\text{cutoff}} U r^2 \text{d}r \text{d}A \cos \theta \text{d}r \]

\[ = dA \text{d}A \text{d}t \frac{UC \cos \theta}{4\pi} \]

\[ : I(\theta) = \frac{UC}{4\pi} \cos \theta \]
3b. Yes, it is a Lambert emitter, since
\[ I \propto \cos \theta. \]

3c. \[
\frac{U}{4\pi} \cos \theta \, d\Omega = \frac{\pi}{2} \int_0^{2\pi} \frac{U}{4\pi} \cos \theta \, \sin \theta \, d\theta \, d\phi = \frac{\pi}{2} \left( \frac{U}{4\pi} \sin \theta \right) \int_0^{2\pi} \sin \theta \, d\phi
\]
\[
= \frac{2\pi}{4\pi} \left( \frac{U}{4\pi} \sin \theta \right) = \frac{2\pi}{4\pi} \left( \frac{U}{4\pi} \sin \theta \right)
\]

Let \( x = \sin \theta \), \( \Rightarrow x = 2\pi \left( \frac{U}{4\pi} \right) \int_0^{1} x \, dx = 2\pi \left( \frac{U}{4\pi} \right) \frac{1}{2} \frac{x^2}{2} = \frac{1}{2} \cdot 2\pi \cdot \frac{U}{4\pi}
\]
\[
= \frac{UC}{4} \, \text{erg/s/cm}^2 \cdot \text{s}. \quad \text{(Might be worth remembering)}
\]

That \( \langle \cos \theta \rangle_{2\pi} \, \text{steradians} = \frac{1}{2} \)

3d. A disk that just filled the hole \( dA \) above would have just the emitted intensity striking it. This result is independent of the location and orientation of the disk inside the box. The received intensity would be calculated per unit area, however, where \( dA' = dA \cos \theta \)

::: \[
\therefore \frac{dI_{\text{received}}}{dt} = \frac{U}{4\pi} \cos \theta \, dA \, d\Omega \, dt
\]

\[ \Rightarrow I_{\text{received}} = \frac{UC}{4\pi} \]

3e. Since we know the hole is a Lambert emitted, the received \( I \) is independent of \( \theta \), and equal to \( UC/4\pi \). This is also what we will get if we integrate the emission from \( dA' \) that will hit \( dA \), where \( dA' \) is the subset of the hole that emits into the cone \( dA \). \( dA' = \frac{dA \, dA'}{\cos \theta} \)