Physics 772 — Problem Set #3

4.1—In astrophysics it is frequently argued that a source of radiation which undergoes a fluctuation of duration $\Delta t$ must have a physical diameter of order $D \lesssim c \Delta t$. This argument is based on the fact that even if all portions of the source undergo a disturbance at the same instant and for an infinitesimal period of time, the resulting signal at the observer will be smeared out over a time interval $\Delta t_{\text{min}} \sim D/c$ because of the finite light travel time across the source. Suppose, however, that the source is an optically thick spherical shell of radius $R(t)$ that is expanding with relativistic velocity $\beta \sim 1, \gamma \gg 1$ and energized by a stationary point at its center. By consideration of relativistic beaming effects show that if the observer sees a fluctuation from the shell of duration $\Delta t$ at time $t$, the source may actually be of radius

$$R < 2\gamma^2 c \Delta t,$$

rather than the much smaller limit given by the nonrelativistic considerations. In the rest frame of the shell surface, each surface element may be treated as an isotropic emitter.

This latter argument has been used to show that the active regions in quasars may be much larger than $c \Delta t \sim 1$ light month across, and thus avoids much energy being crammed into so small a volume.

4.7—An object emits a blob of material at speed $v$ at an angle $\theta$ to the line-of-sight of a distant observer (see Fig. 4.13).

a. Show that the apparent transverse velocity inferred by the observer (i.e., the angular velocity on the sky times the distance to the object) is

$$v_{\text{app}} = \frac{v \sin \theta}{1 - (v/c) \cos \theta}.$$

b. Show that $v_{\text{app}}$ can exceed $c$; find the angle for which $v_{\text{app}}$ is maximum, and show that this maximum is $v_{\max} = \gamma v$.

4.12—Consider a particle of dust orbiting a star in a circular orbit, with velocity $v$. This particle absorbs stellar photons, heats up, and then emits the excess energy isotropically in its rest frame.

a. Show that in absorbing a photon the angular momentum of the particle about the star does not change. (Assume the photons are traveling radially outward from the star.)

b. When the particle emits its radiation, show that the velocity and its direction do not change, but that the angular momentum now decreases by the ratio $m/m'$ of the rest mass after and before emission. Denoting the angular momenta before and after by $l_0$ and $l$, show that

$$l = l_0 \left(1 + \frac{2hv}{mc^2}\right)^{-1/2}.$$

c. Having obtained this general result, let us now assume $v \ll c$ and $hv \ll mc^2$. By expanding, show that to lowest order the change in angular momentum caused by one photon is

$$\Delta l = -\frac{l_0 hv}{mc^2}.$$

Historical note: This result, although now for nonrelativistic particles, apparently cannot be derived classically. Attempts to do so by Poynting and others led to results differing from the correct answer by various numerical factors. Robertson resolved the problem in 1937 (Mon. Not. Roy. Astron. Soc. 97, 423), showing that it is a relativistic effect even to lowest order. The above phenomenon is called the Poynting–Robertson effect.

d. A dust grain having a mass $m \sim 10^{-11}g$ and cross section $\sigma \sim 10^{-8} \text{ cm}^2$ orbits the Sun at 1 A.U. Assuming that it always keeps a circular orbit, find the time for it to fall into the Sun.

4.13 (Optional)

a. Show that an observer moving with respect to a blackbody field of temperature $T$ will see blackbody radiation with a temperature that depends on angle according to

$$T' = \frac{(1 - v^2/c^2)^{1/2}}{1 + (v/c) \cos \theta'} T.$$