Problem 1. Consider the general scattering problem from a potential barrier with classical turning points at \( x = a \) and \( x = b \) (where \( b > a \)). Assuming that the WKB approximation holds, the wavefunction in each region can be written as

\[
\psi_{x<a} = \frac{A}{\sqrt{k(x)}} e^{i \int_a^x k(x') dx'} + \frac{B}{\sqrt{k(x)}} e^{-i \int_a^x k(x') dx'}
\]

\[
\psi_{a<x<b} = \frac{C}{\sqrt{\kappa(x)}} e^{-\int_a^x \kappa(x') dx'} + \frac{D}{\sqrt{\kappa(x)}} e^{i \int_a^x \kappa(x') dx'}
\]

\[
\psi_{x>b} = \frac{F}{\sqrt{k(x)}} e^{i \int_b^x k(x') dx'} + \frac{G}{\sqrt{k(x)}} e^{-i \int_b^x k(x') dx'}
\]

with \( k(x) = \sqrt{2m(E - V(x))/\hbar} \) and \( \kappa(x) = \sqrt{2m(V(x) - E)/\hbar} \). Use the WKB method to compute the \( 2 \times 2 \) matrix \( M \), defined by

\[
\begin{pmatrix} A \\ B \end{pmatrix} = M \begin{pmatrix} F \\ G \end{pmatrix}
\]

in terms of the parameter \( \theta = \exp \left[ \int_a^b \kappa(x') dx' \right] \). For \( G = 0 \) (no incoming wave from the right), write down the transmission coefficient \( T \) in the limit of a high and broad barrier (\( \theta \gg 1 \)).

Problem 2. Consider a particle of energy \( E \) in a one-dimensional symmetric double well potential \( V(x) \) centered at \( x = 0 \), with \( V(x) \to \infty \) for \( x \to \pm \infty \). The classical turning points are \( x = \pm x_1 \) and \( x = \pm x_2 \), with \( x_2 > x_1 \). Here \( E < V_0 \), where \( V_0 = V(0) \) is the maximum height of the central barrier.

Using the WKB method, determine the energy quantization condition for the even and odd parity states in terms of \( \theta = \int_{x_1}^{x_2} k(x') dx' \) and \( \phi = \int_{-x_1}^{x_1} \kappa(x') dx' \). What is the behavior for a high and broad barrier (large \( \phi \))? (Hint: for odd parity states, \( \psi(0) = 0 \), and for even parity states, \( \psi'(0) = 0 \).)

Problem 3. Problem 3.2.

Problem 4. Problem 3.15.

Problem 5. Problem 3.18.
