Problem 1. Let $V$ be the space of real polynomials $p(x)$ defined on the interval $[-\infty, +\infty]$. The inner product is introduced as $\langle p | q \rangle = \int_{-\infty}^{\infty} p(x)q(x) \exp(-x^2)dx$. Starting from the non-orthonormal basis $\alpha_i(x) = x^i$ for $i = 0, 1, \ldots$, use the Gram-Schmidt procedure to construct the first four vectors in an orthonormal basis.

Problem 2. Problem 1.5.

Problem 3. Problem 1.7.

Problem 4. Problem 1.11, modified. A two-state system is characterized by the Hamiltonian

$$H = H_{11} |1\rangle \langle 1| + H_{22} |2\rangle \langle 2| + H_{12} |1\rangle \langle 2| + H_{21} |2\rangle \langle 1|,$$

where $H_{11}$ and $H_{22}$ are real and $H_{12} = S + iA$ with real $S$ and $A$, $|1\rangle$ and $|2\rangle$ form a two-dimensional orthonormal basis. Find the energy eigenkets and the corresponding energy eigenvalues.

Problem 5. Problem 1.19.

Problem 6. Problem 1.20.